"Imagine you're on a train..."

Physics on the Subway

Lee Amosslee
Physics teacher
Carondelet HS; Concord, CA

NSTA
Chicago, IL
November 11, '05
San Francisco: BART (Bay Area Rapid Transit)

- BART serves the San Francisco Bay Area.
- Designed as commuter rail from bedroom communities to cities. The area has since grown to have multiple employment centers, but still most of the ridership is commute into San Francisco.
Today's presentation

1) What physics can you do?

2) Logistics of a subway field trip.

Handouts from today's presentation can be found at www.ShopInBerkeley.com/science or, “Google®” Amosslee Science
Concepts students can investigate

- Acceleration of train
- Force that causes this acceleration
- Frames of reference
- Circular motion (and banking)
**Acceleration**

As trains enter or leave a station, they accelerate with a targeted acceleration.

Too high: people get thrown.

Too low: travel times are longer.

BART's target acceleration: 3 mph/s = 1.34 m/s²
Basic methods for measuring acceleration

– Either:

  ● Measure the distance an object moves in a given amount of time (from rest)

– Or:

  ● Hang a plumb bob and use vectors to determine the acceleration vector compared to gravity.
Distance v time: Rolling a ball

Students need some sort of track to roll the ball along to help keep the ball moving straight and help fix the length.

Upon a signal, one student releases the ball while the other starts the stopwatch. The stopwatch is stopped when the ball reaches the other end.

Students will often think they need to start the observations just as the train starts to accelerate or decelerate, however this is not necessary. It is easier to coordinate timing if one student gives a signal rather than trying to both match the initial “bump” of the train. Ask students why this is so.
Equations:

\[ x = \left( \frac{1}{2} \right) at^2 + v_it + x_i \]

Re-written to solve for acceleration:

\[ a = \frac{(x-x_i -v_it)}{(\frac{1}{2}t^2)} \]

In our “train-ball” system, what we are measuring is the apparent acceleration of the ball, so:

- \( x-x_i \) is the length of the track.
- \( v_i \) = zero (the ball's velocity relative to the train)

Thus:

\[ a = 2 \cdot \text{length} / t^2 \]
Using force to measure acceleration

- Hang a plumb bob and use vectors to determine the acceleration vector compared to gravity.
Using a protractor and a plumb bob, students can record the angle of the bob as the train accelerates.

Students will find angles in the 6-9° range, so their error can be 10-20%.

Discuss with students the advantages and disadvantages of different methods of measuring acceleration.
Forces:
There are two forces on the bob as the train accelerates:

*Gravity* and *the string*.

Since the net acceleration of the train is horizontal, the sum of the force of gravity and the string must be horizontal.

The angle the string show on the protractor is similar to the angle in the vector addition triangle.

\[ F_{\text{net}} = F_{\text{gravity}} \cdot \tan \theta \]
And:
\[ m_{\text{bob}} \cdot a_{\text{net}} = m_{\text{bob}} \cdot a_{\text{gravity}} \cdot \tan \theta \]

Since the mass of the bob appears in both force measurements, it can be cancelled out.

\[ a_{\text{net}} = a_{\text{gravity}} \cdot \tan \theta \]
Three methods to determine acceleration on BART:

1. Rolling a ball and measuring the time to travel a specific distance.
2. Jumping and recording the time in the air and the distance traveled.
3. Hanging a bob and recording the angle from vertical.

Equations:

(I) \[ x = 0.5at^2 + v_i t + x_i \]
Re-written to solve for acceleration:

(II) \[ a = (x-x_i - v_i t) / (0.5t^2) \]
Re-written to solve for \( v_i \):

(III) \[ v_i = (x-x_i - 0.5at^2) / t \]

Rolling a ball
Allow a ball to roll along a track as the car accelerates. Measuring the time it takes to roll (from rest) a measurable distance will provide all the needed variables for equation (II).

Jumping
Jump as high in the air as you can when the train starts to accelerate. Measure the time you are in the air and how far you travel (in the x direction) during your jump, you will have all the variables necessary for equation (II). (Also, calculate your initial jump speed using equation (III)).

Hanging Bob
A bob is a device that hangs due to gravity. Disregard friction, measure the change in the angle \( \theta \). Draw a free body diagram showing the vector for the string’s pull. Since the vector is in the x and (y) direction, you can assume that the resultant vector is in the same direction. Using tangent, write out the equation that uses the gravity (y) and acceleration (net) vectors. If you substitute \( ma \) for F, and 9.8 m/s\(^2 \) for g, you can solve for the acceleration of the train.

The assignment:
You are to determine nine accelerations and nine decelerations of the train as it leaves or enters a station. Use each of the above method three times each for acceleration and deceleration.
<table>
<thead>
<tr>
<th>Row</th>
<th>Acc/Dec</th>
<th>Station</th>
<th>Experiment</th>
<th>Measurements (include units)</th>
<th>Calculations (write out equation with data)</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Positive Acceleration (leaving a station)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Acc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average</td>
</tr>
<tr>
<td>Row</td>
<td>Acc/Dec</td>
<td>Station</td>
<td>Experiment</td>
<td>Measurements (include units)</td>
<td>Calculations (write out equation with data)</td>
<td>Acceleration</td>
</tr>
<tr>
<td>-----</td>
<td>---------</td>
<td>---------</td>
<td>------------</td>
<td>------------------------------</td>
<td>---------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>10</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Dec</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Negative Acceleration (entering a station)**

**Average:**
Frames of reference

The rolling ball experiment lends itself to a discussion of frames of reference:

What we observe in the train is that the ball suddenly starts accelerating. From the frame of reference of the train, the ball has acceleration.

But if we could step outside the train and stand on the ground, we would notice that the ball, once released by the hand, is moving at a constant velocity, and it is the train around it whose speed is changing (where's the force: The wheels are applying a force to the ground).
More Frames of reference

Ask students to observe and discuss relative motion of:

- People on the train vs. people in the station.
- People on the train vs. cars (especially if the train goes down the middle of a road/freeway).
- People on the train vs. other people on the train.
Circular motion and banking

- By measuring the acceleration perpendicular to the motion of the train, students can measure the radius of a curve.
- To calculate the radius of a curve, you will need to know how fast the train is moving.
  - Some trains have a speedometer that is visible from the front car. If you can find the speed of the train:
    - Centripetal acceleration = $v^2/r$
    - Therefore, calculated $a = v^2/r$
- If you can get a map showing the curve, students will see their result is a longer radius curve than reality. This introduces/reinforces the concept that banking a turn decreases the experienced rotational acceleration.
Logistics

- Ask your transit system about student discounts. Many systems charge 16+ year olds full adult fare, but if on a school field trip they can pay youth fare.
- Is there a “reverse commute” direction? If so, start with it (especially for early morning classes).
- Plan, plan, plan. Public transit generally runs on time, but have a back up plan. If you’re planning on going 6 stations out and back, be ready to only 4 or 5.
- Think about which stations have a central platform versus one you have to go up and down stairs. Make it hard for students to get lost.
## Physics on BART Field Trip

**October 26 & 27, 2005**

<table>
<thead>
<tr>
<th></th>
<th>Wednesday, period 1 &amp; 3 (One physics class, period 3)</th>
<th>Thursday, period 2 &amp; 4 (Two physics classes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class size</td>
<td>30</td>
<td>55 (includes DLS boys)</td>
</tr>
<tr>
<td>Planned itinerary</td>
<td>Late (if first BART missed)</td>
<td>Planned itinerary</td>
</tr>
<tr>
<td>Period starts</td>
<td>9:10</td>
<td>8:10</td>
</tr>
<tr>
<td>Leave classroom</td>
<td>9:20</td>
<td>8:20</td>
</tr>
<tr>
<td>Arrive bus stop, NE corner Winton &amp; Treat</td>
<td>9:30</td>
<td>8:30</td>
</tr>
<tr>
<td>Board 115 CC bus (time @ Oak Grove Rd)</td>
<td>9:38</td>
<td>8:38</td>
</tr>
<tr>
<td>Arrive PH BART</td>
<td>9:43</td>
<td>8:43</td>
</tr>
<tr>
<td>Get on P/BP train (14 min, 6 accelerations)</td>
<td>9:49</td>
<td>8:49</td>
</tr>
<tr>
<td>Arrive P/BP</td>
<td>10:03</td>
<td></td>
</tr>
<tr>
<td>Exit train at NC/M if late</td>
<td>10:12</td>
<td>9:12</td>
</tr>
<tr>
<td></td>
<td>14 minute wait for train change, same platform</td>
<td></td>
</tr>
<tr>
<td>Leave P/BP (32 min, 14 accelerations tunnel slope calculation)</td>
<td>10:17</td>
<td>9:17</td>
</tr>
<tr>
<td>Leave NC/M if late</td>
<td>10:23</td>
<td>9:23</td>
</tr>
<tr>
<td>Arrive Rockridge BART</td>
<td>10:49</td>
<td>9:49</td>
</tr>
<tr>
<td></td>
<td>12 minute wait for train change, same platform</td>
<td></td>
</tr>
</tbody>
</table>